

CMB Aberration and Doppler Effects as a Source of Hemispherical Asymmetries

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Our peculiar motion with respect to the CMB rest frame represents a preferred direction in the observed CMB sky since it induces an apparent deflection of the observed CMB photons (aberration) and a shift in their frequency (Doppler). Both effects distort the multipoles $a_{\ell m}$'s at all ℓ 's. Such effects are real as it has been recently measured for the first time by [1] according to what was proposed in [2–4]. However, the common lore when estimating a power spectrum from CMB is to consider that Doppler affects only the $\ell = 1$ multipole, neglecting any other corrections. In this letter we use simulations of the CMB sky in a boosted frame with a peculiar velocity $\beta \equiv v/c = 1.23 \times 10^{-3}$ in order to assess the impact of such effect on power spectrum estimations in different regions of the sky. We show that the boost induces a north-south asymmetry in the power spectrum which is highly significant and non-negligible, of about $(0.75 \pm 0.15)\%$ for half-sky cuts when going up to $\ell \approx 2500$. We suggest that these effects are relevant and may account for some of the north-south asymmetries seen in the Planck data, being especially important at small scales.

Keywords: CMB theory, CMB aberration, CMB anomalies

Introduction. The Cosmic Microwave Background is used as a fundamental tool to test Cosmological Models and to quantitatively estimate the parameters of such models. This is usually done by extracting the Temperature and the Polarization power spectra from the maps and by fitting them with a Λ CDM background model with an almost scale-invariant gaussian spectrum of density perturbations. However, if we observe the CMB with a velocity $\beta \equiv v/c$ relative to such background, the image undergoes distortions due to the Doppler and aberration effects and the correct procedure should be to first transform the image in the CMB rest frame and then analyze the data. Surprisingly such procedure is not performed by any of the current experimental analysis.

Such boost in a direction defined by the unit vector $\hat{\mathbf{z}}$ distorts a primordial temperature map $T(\hat{\mathbf{n}})$ by a Doppler factor and by changing the arrival direction:

$$T'(\hat{\mathbf{n}}') = \gamma(1 + \beta \hat{\mathbf{n}} \cdot \hat{\mathbf{z}})T(\hat{\mathbf{n}}), \quad (1)$$

where $(\hat{\mathbf{n}}' - \hat{\mathbf{n}}) \cdot \hat{\mathbf{z}} = \beta \sin^2 \theta / (1 + \beta \cos \theta)$, $\cos \theta = \hat{\mathbf{n}} \cdot \hat{\mathbf{z}}$ and $\gamma \equiv 1/\sqrt{1 - \beta^2}$. Such distortion introduces correlations between different $a_{\ell m}$'s (the coefficients of the spherical harmonics decomposition), both *diagonally* (i.e., between same ℓ 's) and *off-diagonally* (between different ℓ 's) [5]:

$$a_{\ell m}^{[\text{Boosted}]} = \sum_{\ell'} K_{\ell' \ell m} a_{\ell' m}^{[\text{Primordial}]}. \quad (2)$$

The only effect which is usually taken into account is the large $\ell = 1$ dipole, due to the matrix element K_{010} , which is used to infer our velocity β [6]. Clearly however there is much more information in the other matrix elements. The off-diagonal correlation was shown in [2–4] to be measurable by Planck as an alternative method to measure β and in fact the Planck collabora-

tion itself [1] recently published for the first time a detection of β through CMB aberration and Doppler using $1 < \ell \lesssim 2000$. This provides an alternative measurement of β with respect to the usual $\ell = 1$ Doppler effect. For the diagonal correlation it has been shown in a previous paper [7] (see also [8] for earlier work) with numerical simulations of the CMB sky that while in the full-sky case it is safe to disregard such boost effects, there is instead a non-negligible distortion in power spectrum reconstruction due to the presence of a mask which may induce a significant bias in the parameter estimation for Planck, up to 1 – 2 standard deviations.

In this letter we assess whether ignoring such frame effect may also induce a significant spurious directional dependence in the CMB power spectrum. In particular we analyze the pseudo- C_ℓ reconstructed in different regions of the sky by performing simulations in the case in which a boost effect is present or not. We proceed here as in [7]: we simulate maps of the CMB sky and we directly apply on the maps the boost transformation before extracting the $a_{\ell m}$'s, therefore bypassing the need of computing the mixing coefficients. Then we extract the $a_{\ell m}$'s and the C_ℓ 's and show that an asymmetry is visible. We also roughly quantify the amount of asymmetry through an asymmetry coefficient which is built to give an estimate of the change of the overall amplitude of the power spectrum. We apply all this procedure to Temperature maps for an ideal experiment (neglecting instrumental noise) for 3 different symmetric sky-cuts with measurable fraction of the sky $f_{\text{sky}} = \{0.88, 0.29, 0.04\}$) to a maximum multipole $\ell_{\text{max}} = 2700$. Our results can thus easily represent a Planck-like case (almost all-sky and $\ell_{\text{max}} \simeq 2500$), a WMAP-like case (almost all-sky and $\ell_{\text{max}} \simeq 900$) and in part some other current small-scale surveys, like SPT [9] and ACT [10] (smaller f_{sky}

and $\ell_{\max} \simeq 3000$).

Power spectra asymmetry. We apply our procedure to extract pseudo- C_ℓ 's from different regions of the sky in simulations performed with the HEALPix package¹ in a modified version [7] which allows boosts, with $N_{\text{side}} = 4096$ and $\ell_{\max} = 2700$. Such a numerical boost procedure has been tested with Bessel fitting functions which reproduce the $K_{\ell'\ell m}$ with high precision [4, 7].² We apply a boost along the north pole direction of $\beta = 0.00123$ and compare two regions in two opposite directions along such boost (dubbed North and South). We performed 20 different simulations as random realizations, labelled by the random seed used as an input in HEALPix, of a fiducial cosmological model similar to the WMAP 9-year best fit. We find that there is a systematic asymmetry between power in the two opposite directions in the case with boost as opposed to the case without boost. We show the difference in power spectra in Fig. 1. This should be compared with the experimental results presented in [12] (section 5.5.1 and Fig. 28), from which we can see that the real data has systematically more power in one hemisphere than the other at a level of a few percent, although it is difficult to make a more precise estimate on the significance and scale of the effect based on the results presented at that paper.

Given a set of C_ℓ 's it is possible to have a rough estimate on the size of the effect on a cosmological parameter, by considering [7] an idealized case in which the CMB depends multiplicatively on a single amplitude parameter, which we call A , so that the χ^2 is given by:

$$\chi^2(A) = \sum_{\ell} \frac{(C_{\ell}^{\text{exp}} - A\hat{C}_{\ell}^{\text{th}})^2}{\sigma_{\ell}^2}, \quad (3)$$

where $\hat{C}_{\ell}^{\text{th}}$ is the theoretical spectrum when $A = 1$, C_{ℓ}^{exp} are the observed values in one region of the sky and where $\sigma_{\ell}^2 = C_{\ell}^2 2/(2\ell + 1)$ is the cosmic variance, ignoring any noise. The best fit value A_{bf} for A is obtained when $\partial(\chi^2)/\partial A = 0$ which gives:

$$A_{\text{bf}} = \sum_{\ell} \frac{C_{\ell}^{\text{exp}} \hat{C}_{\ell}^{\text{th}}}{\sigma_{\ell}^2} / \sum_{\ell} \frac{(\hat{C}_{\ell}^{\text{th}})^2}{\sigma_{\ell}^2}, \quad (4)$$

The difference $A_{\text{bf}}^N - A_{\text{bf}}^S$, between the best-fit values in two regions N and S of the sky with observed spectra $C_{\ell}^{\text{exp}, N}$ and $C_{\ell}^{\text{exp}, S}$ is therefore given by:

$$\frac{\delta A}{A} \equiv 2 \frac{A_{\text{bf}}^N - A_{\text{bf}}^S}{A_{\text{bf}}^N + A_{\text{bf}}^S} \simeq \left(\sum_{\ell} \frac{\delta C_{\ell}}{C_{\ell}} (2\ell + 1) \right) / \sum_{\ell} (2\ell + 1), \quad (5)$$

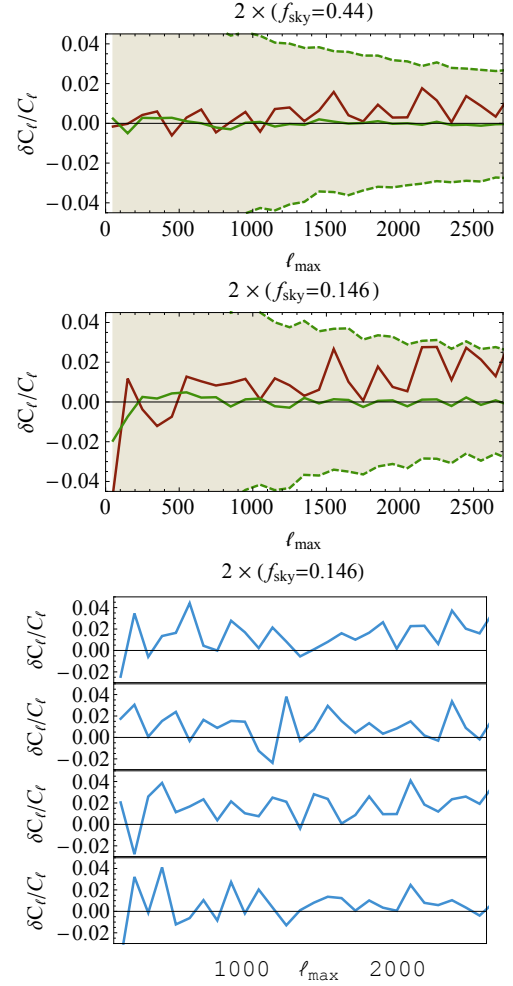


Figure 1. Relative difference between C_ℓ 's in two opposite discs of the sky centered on the dipole direction as a function of the multipole ℓ . The brown (green) line shows the mean spectrum over 20 boosted (unboosted) simulations, binned in 100- ℓ bins. The green area shows the 1σ band around the unboosted mean. [Top]: two halves of the sky ($f_{\text{sky}} = 0.44$, after removing a band around the galaxy). [Middle]: two antipodal discs of 90° diameter ($f_{\text{sky}} = 0.146$, to be compared with [12]). [Bottom]: 4 random boosted realizations.

where $\delta C_\ell \equiv C_{\ell}^{\text{exp}, N} - C_{\ell}^{\text{exp}, S}$ and we have approximated C_{ℓ}^{th} by the average $(C_{\ell}^{\text{exp}, N} + C_{\ell}^{\text{exp}, S})/2$. We therefore estimate³ $\delta A/A$ for our simulations summing up to a certain ℓ_{\max} . We will compute the estimator directly using the pseudo- C_ℓ 's given as an output by HEALPix in eq.(5). For a sufficiently large patch of the sky the offset between pseudo- C_ℓ and C_ℓ is simply an overall factor given by the sky fraction f_{sky} so our eq.(5) is still a good estimator of the difference in amplitude of the two hemispheres. Note however that we will also use the same

¹ <http://healpix.sourceforge.net/>

² We have checked up to $\ell_{\max} = 3000$ that boosting a $N_{\text{side}} = 4096$ map with $\beta = 0.00123$ and boosting again with $-\beta$ gives back the original map with very high precision up to the 7th digit in the $a_{\ell m}$'s, differently from what obtained in [11].

³ Such an estimator has been used also by [13] to quantify hemispherical asymmetries.

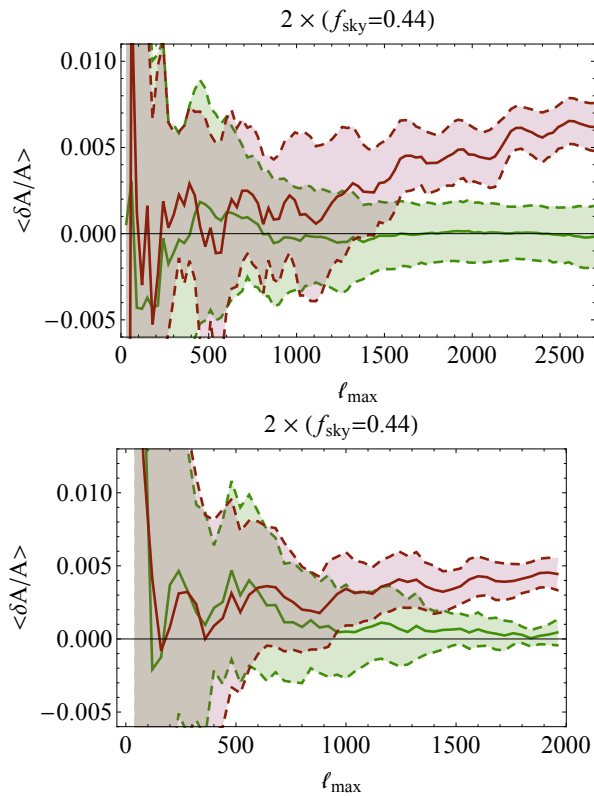


Figure 2. Average and 1σ bands for 20 realizations of the North-South asymmetry parameter $\delta A/A$ of eq.(5). Brown curves are made on boosted maps, green on non-boosted ones. The difference is computed using pseudo- C_ℓ 's in two halves of the sky ($f_{\text{sky}} = 0.44$, after removing a band around the galaxy). [Top] boost towards the north pole. [Bottom]: boost along the actual direction given by the measured $\ell = 1$ CMB dipole, $[(l, b) = (264, 48)]$ in galactic coordinates.

equation for smaller sky patches with the *caveat* that in this case $\delta A/A$ is not directly related to the change in amplitude (since the pseudo- C_ℓ and the C_ℓ are related by a window function [14] and also eq.(3) should have a full covariance matrix) but still represents a simple meaningful way to quantify an asymmetry. The typical values we obtain for $\delta A/A$ are small and centered around zero for the case without aberration while they represent an important effect of order 1% when aberration is present.

We show in Figs. 2 and 3 the result for mean and standard deviation of the asymmetry parameter as a function of ℓ_{\max} for our 20 simulations. Such an asymmetry is definitely detectable: at $\ell_{\max} \approx 2000$, it amounts to 3σ for $f_{\text{sky}} = 2 \times 0.44$ and to over 3.5σ for $f_{\text{sky}} = 2 \times 0.146$, and the significance increase with ℓ_{\max} (e.g. it is above 5σ at $\ell_{\max} \approx 2500$). Actually such an asymmetry in the power spectrum constitutes by itself a self-consistency check on the measurement of β given in [1], as already proposed initially by [15]. Given such a large result it is thus natural to wonder whether this could account for the asymmetries measured by Planck at $2 < \ell < 1500$, or at least for a

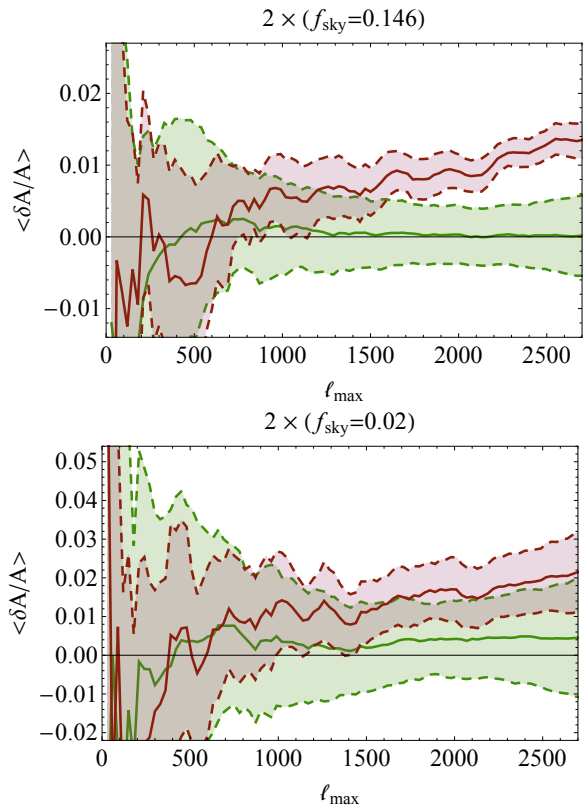


Figure 3. Same as Fig. 2 for smaller patches of the sky. Note that although the discrepancy is larger for smaller f_{sky} , this is over-compensated by the increase of variance among different realizations. Nevertheless the observed CMB is one particular realization and so the difference on a parameter estimation may well be large (of order a few percent). [Top]: two antipodal discs of 90° diameter ($f_{\text{sky}} = 0.146$, similarly to the very recent Planck collaboration asymmetry analysis (Fig. 28 in [12])). [Bottom]: two antipodal discs of 32.5° diameter ($f_{\text{sky}} = 0.02$, similar to the partial SPT survey).

substantial fraction of it. Looking more closely at the results of [12], sect. 5.5.1 we can see that Planck detects an overall preferred direction with a significance quantified by the fraction of simulations that have smaller clustering of the dipole directions than the data: such number is of about 11/500, 4/500 or less than 1/500, depending on the foreground cleaning methods, which corresponds to a 97% – 99% C.L. When looking at our Figs. 2 and 3 at $\ell = 1500$ we can see that a $1.5 - 2\sigma$ effect could be easily achieved and it may therefore happen that, once the boost is subtracted, the Planck anomalies could become much less significant or in any case considerably change. There is some small effect already at $\ell_{\max} \simeq 600 - 900$, which should be compared with the asymmetries seen in the WMAP experiment at $2 < \ell < 600$ by [16–18], see also [19]. In this case the claimed significance was of about 99% C.L. [16–18] and again it is important reassess such significance once the boost is subtracted. It is also interesting to note that the preferred direction

of WMAP in [16–18] does not quite point in the same direction as the dipole but it is not very far away from it. More interestingly, for Planck the overall direction of the asymmetry is still close to WMAP but it is a bit shifted towards the dipole direction. In fact, when looking at Fig. 27 of [12], which shows the preferred direction when analyzing the data in $100 - \ell$ bins, we can see that most bins, especially at low ℓ , point towards the same direction of WMAP but there are several bins which point instead to the dipole direction, especially at large ℓ . This seems consistent with our findings that at least a sizable fraction of the asymmetry may be due to our motion in the direction of the dipole, especially at large ℓ . Note instead that asymmetries and anomalies at very large scales $\ell \lesssim 60$ [16, 17] cannot be accounted for by a boost alone. It is reasonable therefore to consider the possibility that there may be some intrinsic large scale asymmetry pointing to a different direction which adds up with our boost effect. This hypothesis deserves a more thorough analysis of the real data, going also to $\ell > 1500$, and crucially stresses the need of analyzing the WMAP and especially Planck data by first removing the aberration and Doppler effects (from the whole map, not only the dipole) and then looking for the true significance of eventual residual anomalies.

Fig. 2 (bottom panel) shows the result in the case in which the hemispheres are not aligned with the dipole. This was achieved by boosting the map along the North-South axis and then rotating it with HEALPix along the actual dipole direction $[(l, b) = (264, 48)]$ in galactic coordinates as measured by the $\ell = 1$ multipole of the CMB by WMAP and then performing the cut around the North and South directions. The results we find for the asymmetry parameter $\delta A/A$ is of lower size and significance, as is to be expected since the North-South direction now is not anymore the one which maximizes the asymmetry. Finally Fig. 3 focuses on the case of smaller discs, the first of which allows a direct comparison with [12] (it corresponds to two antipodal 90° -diameter discs). Note that in these latter cases the size of the effect is larger, because we are selecting the regions of the sky most affected by a boost (small discs around the poles), but at the same time the variance is bigger because we are sampling a small fraction of the sky. The bottom panel depicts small area analysis that would be relevant for experiments such as SPT [9] and ACT [10] which look at small regions (although their actual observation area are not circular nor antipodal) of the sky and are therefore subject to a potentially large bias due to the boost. We leave the analysis of such bias for future work.

Conclusions. In this letter we have considered the effect of our peculiar velocity on the CMB power spectra in regions in opposite directions of the sky. As in [7], we have applied a boost transformation (1) to this frame directly in pixel space on simulated maps, rather than on the $a_{\ell m}$'s, which yields a direct test (although

a similar result could be obtained with the fitting functions in [4]). We find that aberration and Doppler effects induce a directional dependence of the amplitude of the power spectrum in opposite directions which we estimate to be equal to $(0.75 \pm 0.15)\%$ for half sky cuts when summing up to $\ell \lesssim 2500$ and is therefore highly significant. We claim this could be important to understand the Hemispherical asymmetry found by Planck [12]; at least it must account for an important fraction of it. It may have also some impact on previous detections in the WMAP data at $2 < \ell < 600$ [16–18], but by itself it cannot explain an anomaly localized only at low- ℓ (say, $\ell \leq 60$ as in [20, 21]). Note that the direction of maximal asymmetry detected in [12] and [16–18] does not coincide with the dipole direction, but some of the bins used for the analysis in [12] do indeed point towards the dipole direction: this suggests that the asymmetries reported by Planck are likely to be a mixture of a boost effect, showing up especially at high ℓ , with some residual asymmetry mostly relevant at low ℓ . It would be useful in this respect to analyze the real Planck data at higher ℓ to check whether the asymmetry converges towards the dipole direction. Note that we are relying on the standard assumption that our velocity is given by the CMB dipole; if that is not the case there might be a discrepancy between the dipole and aberrated directions. In any case our findings clearly indicate that before looking for preferred directions in the real data at high multipoles one should properly deboost the map going to the CMB rest frame to avoid spurious detection of anomalies.

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